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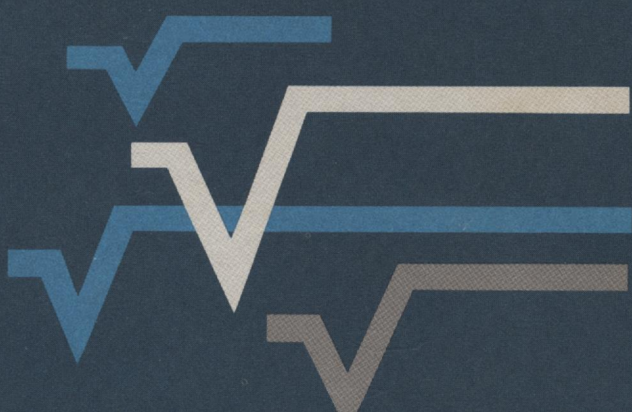
simplified methods for extracting roots



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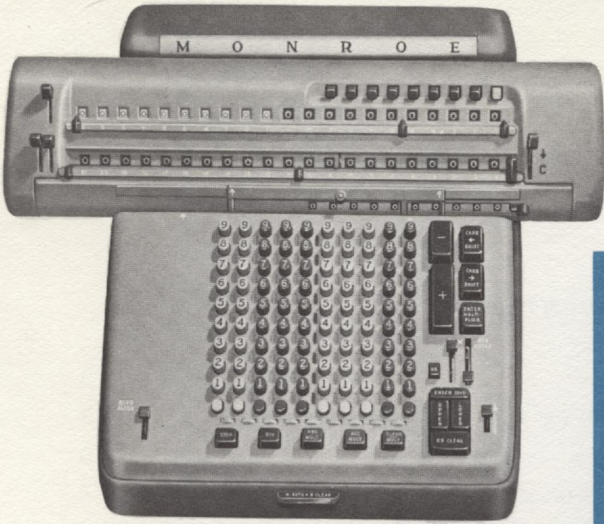

introduction

This book has been prepared by Monroe to simplify the extraction of roots. Credit for its completion is given to Norman Wyman Storer, Department of Physics and Astronomy, The University of Kansas for his Table of Factors for the Extraction of Square Roots. George E. Reynolds, Electromagnetic Radiation Laboratory, Air Force Cambridge Research Center developed the Five-Place Cube Root Table reproduced herein.

SIMPLIFIED MONROE TABLE OF DIVIDING FACTORS FOR SQUARE ROOTS

The simplified Monroe table of dividing factors for square roots gives accuracy to five significant figures, with an error less than 5 in the sixth figure.

instructions

To find $\sqrt{N}$, first determine N′ as follows:

1. For N between 1 and 100 inclusive, take $N' = N$.
2. For N less than 1 or greater than 100, move the decimal point to the right or left in steps of two digits to arrive at N′ between 1 and 100.

Find the two consecutive values in the n′ column between which N′ lies, and select the values of A and D between the two selected n′ values.

Monroe method

1. Set N′ on the extreme left of the keyboard and enter as a dividend.
2. Set A on the extreme left of the keyboard and add.
3. Set D on the extreme left of the keyboard and divide.

The result in the upper dials of the Monroe after the decimal point is inserted is the square root with an error less than 5 in the sixth digit.

pointing off decimals in roots

If N is greater than 1, start at the decimal point and working to the left, set off N into groups of two digits each. The number of such two digit groups to the left of the decimal point will be the number of digits to the left of the decimal point in the root. If the extreme left-hand group consists of only one figure, it should be counted as though a complete group.

If N is less than 1, start at the decimal point and working to the right, set off the zeros preceding the first significant figure into groups of two zeros each. The number of such groups will be the number of zeros that should follow the decimal point and precede the first significant figure in the root. If the last right-hand group consists of only one zero, it should NOT be counted as a group.

EXAMPLE I

Evaluate $\sqrt{6942.3214}$

Determine $N' = 69.423214$, $A = 69889$ and $D = 1672$.

1. Set 69423214 on the extreme left of the keyboard and enter in the lower dials as a dividend.
2. Set 69889 on the extreme left of the keyboard and add.
3. Set 1672 on the extreme left of the keyboard and divide.

Read in the upper dials 833207 and round off to 83321. Proper insertion of the decimal point gives the root, 83.321. The decimal point in the root is found by setting off the whole number 6942 into groups of two digits each, 69'42. Since there are two groups, there are according to the rule two whole number digits in the root, thus 83.321.

EXAMPLE II

Evaluate $\sqrt{0.000003912}$

Determine $N' = 3.912$, $A = 3863$ and $D = 3931$.

1. Set 3912 on the extreme left of the keyboard and enter in the lower dials as a dividend.
2. Set 3863 on the extreme left of the keyboard and add.
3. Set 3931 on the extreme left of the keyboard and divide.

Read in the upper dials 197786 and round off to 19779. Proper insertion of the decimal point gives the root 0.0019779. The decimal point is found by counting the number of full pairs of zeros to the immediate right of the decimal point in 0.000003912. Since there are two such pairs of zeros (disregard the fifth zero) two zeros should follow the decimal point and precede the first significant figure of the root, thus 0.0019779.

EXAMPLE III

Evaluate $\sqrt{730.6789}$

Root required to ten significant figures.

A Monroe model with ten columns on the keyboard is required to secure the root to ten significant figures.

Following the same procedure as in Examples 1 and 2, find $\sqrt{730.6789} = 27.0310$, a result with an error less than 5 in the sixth place. Up to this point a Monroe with an eight column keyboard can be used. To secure the root to ten significant figures and using a ten column model, proceed from this point as follows:

1. Divide 7306789 by 27031 to obtain a ten place answer 2703114572, disregarding decimal points.
2. Average 2703114572 with our first approximation, 270310, i.e., add 2703114572 and 270310 with the left-hand digits aligned, to obtain 5406214572, and divide this figure by 2 to obtain the ten place result 2703107286. When the decimal is pointed off, we obtain 27.03107286, the result with a maximum possible error of 1 in the tenth place.

Following this method an eight-column Monroe can be used to find the root to eight places.

SQUARE ROOT TABLE

n′	A	D
1.000	102	202
1.045	1067	2066
1.09	1114	2111
1.14	1162	2156
1.19	1211	2201
1.24	126	2245
1.28	1303	2283
1.327	1356	2329
1.385	1416	238
1.45	1481	2434
1.515	1545	2486
1.575	1609	2537
1.645	1677	259
1.705	1741	2639
1.777	1817	2696
1.855	1896	2754
1.94	1974	281
2.01	2042	2858
2.08	2114	2908
2.16	2177	2951
2.222	2259	3006
2.29	2327	3051
2.38	2421	3112
2.47	252	3175
2.57	2621	3238
2.676	2729	3304
2.78	2829	3364
2.885	2936	3427
2.99	3038	3486
3.09	3154	3552
3.22	3285	3625
3.352	3404	369
3.457	3523	3754
3.59	3648	382
3.71	3777	3887
3.80	3863	3931
3.94	3996	3998
4.05	4056	4028
4.13	4188	4093
4.26	432	4157
4.40	4471	4229
4.54	4618	4298
4.705	4785	4375
4.87	4946	4448
5.035	513	453
5.225	5306	4607
5.39	5459	4673
5.56	565	4754
5.74	5832	483
5.93	6017	4906
6.12	62	498
6.29		

n′	A	D
6.29	6383	5053
6.49	6579	513
6.69	6791	5212
6.905	7017	5298
7.132	7244	5383
7.35	745	5459
7.56	7681	5543
7.80	7924	563
8.05	8168	5716
8.285	8404	5798
8.53	8661	5886
8.795	8922	5974
9.00	9102	6034
9.24	9345	6114
9.485	9619	6203
9.77	9894	6291
10.0	1009	6353
10.2	1032	6425
10.45	1053	649
10.6	1066	653
10.8	1086	6591
11.0	11122	667
11.2	1136	6741
11.5	1157	6803
11.7	1183	6879
11.9	1202	6934
12.1	1218	698
12.3	1245	7057
12.5	1262	7105
12.75	1287	7175
12.9	1305	7225
13.2	13286	729
13.4	1352	7354
13.7	1379	7427
13.9	1401	7486
14.1	1419	7534
14.3	1436	7579
14.5	146	7642
14.7	1488	7715
15.08	1528	7818
15.45	1563	7907
15.8	1596	799
16.0	1616	804
16.35	1652	8129
16.7	169	8222
17.1	1721	8297
17.35	1756	8381
17.75	1788	8457
18.05	1819	853
18.4	1858	8621
18.8	1897	8711
19.2		

n′	A	D
19.2	1932	8791
19.55	1978	8895
20.0	2016	898
20.3	2043	904
20.65	2088	9139
21.0	211	9187
21.3	2152	9278
21.7	2186	9351
22.1	2233	9451
22.5	227	9529
22.95	2316	9625
23.3	2343	9681
23.6	2379	9755
24.0	2427	9853
24.55	248	996
25.0	252	1004
25.5	2574	10147
26.0	2626	10249
26.55	26832	1036
27.1	2741	10471
27.72	27984	1058
28.25	2851	10679
28.8	29052	1078
29.3	2955	10872
29.8	3003	1096
30.2	3047	1104
30.8	31136	1116
31.45	31753	1127
32.05	3237	11379
32.7	33062	115
33.44	33698	1161
33.9	34222	117
34.6	3491	11817
35.2	35402	119
35.78	3606	1201
36.3	36602	121
37.0	37271	1221
37.5	37822	123
38.2	38502	1241
38.8	39062	125
39.4	3974	12608
40.0	40322	127
40.7	4103	12811
41.45	41796	1293
42.1	4251	1304
42.9	4323	1315
43.7	4389	1325
44.3	44689	1337
45.0	4536	1347
45.7	4609	13578
46.5	46922	137
47.35		

n′	A	D
47.35	47679	1381
48.0	48302	139
48.75	4914	1402
49.5	4985	14121
50.35	5073	14245
51.1	5148	1435
51.8	522	1445
52.7	5318	14585
53.6	54022	147
54.5	54908	1482
55.2	55502	149
56.0	564	1502
56.83	5738	1515
57.9	5843	15288
59.0	59444	1542
59.85	6038	15541
60.95	61465	1568
62.0	6252	15814
63.1	636	1595
64.0	644	1605
65.0	6548	16184
66.0	66585	1632
67.2	6765	1645
68.25	6879	16588
69.3	69889	1672
70.4	7098	1685
71.4	7191	1696
72.5	73102	171
73.73	7439	1725
75.0	7549	17377
76.0	76562	175
77.2	7788	1765
78.5	79032	1778
79.5	80102	179
80.6	8109	1801
81.6	8221	18134
82.8	8321	18244
83.7	8418	1835
84.6	851	1845
85.5	8584	1853
86.4	8689	18643
87.4	8789	1875
88.3	8883	1885
89.4	8968	1894
90.0	9044	1902
91.0	9168	1915
92.1	9264	1925
93.3	9406	19397
94.6	9516	1951
95.75	9653	1965
97.1	9792	19791
98.6	994	1994
100.0		

INTRODUCTION TO
EXTRACTION OF CUBE ROOTS

The simplicity of this new method for extracting cube roots on Monroe 8N-213 and other models, with the aid of a brief table, is such that it can be mastered easily. The use of this table is advantageous in that it reduces laborious and complicated calculations to a simple process of multiplication and division, and provides accuracy of five significant places.

Following the five-place cube root table is a method of extending any cube root to an accuracy of ten significant places.

Monroe method

PROGRAM

Decimals:		
Upper dials	5	Tab at 6
Keyboard	5	Auto. KB Clear to the right
Lower dials	10	

Step 1. Adjust the decimal point in the number whose cube root is desired so that there are three whole number places. Enter the adjusted number as a dividend.

Step 2. Divide by the number, in the column of approximations in the table, which is nearest to the adjusted number, but not greater than it.

Step 3. Select the multiplier in line with the approximation used corresponding to the range of the column which will contain the original number. If the original number is not contained in one of the given ranges, then shift the decimal point, either left or right, three places at a time until the number does fall within one of the ranges. Clear the keyboard and enter the proper multiplier.

Step 4. Copy the quotient (of which the first and second digits are always one and zero, respectively) from the upper dials to the keyboard, changing the second digit (zero) to a five and then multiply.

Step 5. Change the first and second digits of the keyboard quantity (one and five, respectively) to a three and zero, respectively. Divide, and the quotient is the desired cube root. If any shifting of the decimal point took place in step 3 to determine the proper multiplier, the decimal point in the result must be moved one place in the opposite direction of each three-place shift.

In some cases there may be an error of one unit in the fifth significant place due to round-off of the final result.

EXAMPLE I

Find the $\sqrt[3]{27.634}$

PROGRAM

Decimals:

Upper dials	5	Tab at 6
Keyboard	5	Auto. KB Clear to the right
Lower dials	10	

Step 1. Enter the dividend 276.34.

Step 2. Divide by 260. Result in the upper dials 1.06284.

Step 3. Clear the keyboard and select the proper multiplier.

Step 4. Multiply 5.9250 by 1.56284. Result in the lower dials is 9.259827.

Step 5. Divide by 3.06284. Result in the upper dials is 3.02328.

$$\sqrt[3]{27.634} = 3.0233$$

EXAMPLE II

Find the $\sqrt[3]{168498.}$

PROGRAM

Decimals:

Upper dials	5	Tab at 6
Keyboard	5	Auto. KB Clear to the right
Lower dials	10	

Step 1. Enter the dividend 168.498.

Step 2. Divide by 162. Result in the upper dials is 1.04011.

Step 3. Clear the keyboard and select the proper multiplier.

Step 4. Multiply 10.9028 by 1.54011. Result in the lower dials is 16.791511308.

Step 5. Divide by 3.04011. Result in the upper dials is 5.52332.

Because of the necessary one three-place shift to the left in step 3, the decimal point in the result must be moved one place to the right.

$$\sqrt[3]{168498.} = 55.233$$

EXAMPLE III

Find the $\sqrt[3]{.00000489}$

PROGRAM

Decimals:

Upper dials	5	Tab at 6
Keyboard	5	Auto KB Clear to the right
Lower dials	10	

Step 1. Enter the dividend 489.

Step 2. Divide by 464. Result in the upper dials is 1.05387.

Step 3. Clear the keyboard and select the proper multiplier.

Step 4. Multiply 3.3359 by $1.\underline{5}5387$. Result in the lower dials is 5.183554933.

Step 5. Divide by $\underline{3.0}5387$. Result in the upper dials is 1.69737.

Because of the necessary two three-place shifts to the right in step 3, the decimal point in the result must be moved two places to the left.

$$\sqrt[3]{.00000489} = .016974$$

FIVE-PLACE CUBE ROOT TABLE

APPROXIMATIONS TO THE NUMBER WHOSE CUBE ROOT IS DESIRED	Multipliers 1 to 10	10 to 100	100 to 1000
100	2.0000	4.3089	9.2832
107	2.0457	4.4072	9.4950
115	2.0954	4.5144	9.7259
123	2.1429	4.6168	9.9464
132	2.1940	4.7267	10.1833
141	2.2427	4.8318	10.4097
151	2.2946	4.9434	10.6502
162	2.3490	5.0606	10.9028
173	2.4010	5.1727	11.1442
185	2.4553	5.2896	11.3961
198	2.5115	5.4107	11.6570
212	2.5693	5.5354	11.9255
227	2.6285	5.6629	12.2004
243	2.6889	5.7930	12.4806
260	2.7502	5.9250	12.7651
278	2.8122	6.0587	13.0531
297	2.8749	6.1937	13.3439
317	2.9380	6.3297	13.6370
338	3.0015	6.4665	13.9317
360	3.0653	6.6039	14.2276
384	3.1319	6.7475	14.5370
409	3.1985	6.8909	14.8459
436	3.2674	7.0393	15.1656
464	3.3359	7.1869	15.4836
494	3.4063	7.3385	15.8103
526	3.4783	7.4937	16.1446
560	3.5517	7.6518	16.4852
595	3.6242	7.8080	16.8217
632	3.6978	7.9666	17.1634
672	3.7742	8.1312	17.5181
714	3.8512	8.2972	17.8757
758	3.9288	8.4643	18.2356
805	4.0084	8.6357	18.6050
854	4.0881	8.8075	18.9751
906	4.1694	8.9827	19.3527
961	4.2522	9.1609	19.7366

EXTENSION OF ACCURACY

Any cube root may be extended to ten significant places if so desired.

A handy reference during the solution of an extension would be to set up this table on paper:

$N =$ The original number whose cube root was desired.

$X^2 =$ Determined in step 1 of the extension.

$X =$ The cube root of N that is to be extended.

PROGRAM

Decimals:	No tab
None	Repeat on

All numbers are entered on the left of the keyboard.

Step 1. Square X and copy the result to the table.

Step 2. Enter the dividend N.

Step 3. Divide by X^2.

Step 4. Enter the upper dial number as a dividend.

Step 5. Multiply accumulatively 2 by X.

Step 6. Divide by 3.

The upper dials contain the cube root extended to ten significant places. Place the decimal point as it appears in the original X.

EXAMPLE OF EXTENSION OF ACCURACY

Extend the accuracy of the cube root of Example 2 to ten significant places.

Table: $N = 168498.$

$X^2 = 3050684289$

$X = 55.233$

PROGRAM

Decimals:	No tab
None	Repeat on

All numbers are entered on the left of the keyboard.

Step 1. Square 55233. Copy lower dials to the table.

Step 2. Enter the dividend 168498.

Step 3. Divide by 3050684289. Result in the upper dials is 55232854.

Step 4. Enter the dividend 55232854.

Step 5. Multiply accumulatively 2 by 55233. Result in the lower dials is 165698854.

Step 6. Divide by 3. Result in the upper dials is 5523295133.

$\sqrt[3]{168498.} = 55.233 = 55.23295133$ (extended to ten significant places)

EXTRACTION OF HIGHER ROOTS

outline of method

In starting the extraction of a higher root refer to the table of approximations to determine the initial approximation. Reading down in the desired root column find the range that contains the number whose root is wanted. In line with this range in the extreme left column is the initial approximation.

The machine operations consist of using this initial approximation in the formula given below to calculate a second approximation. The second approximation is then substituted in the same formula to obtain a third approximation. This process is repeated until two approximations coincide. The last approximation will be the correct root of the number.

If the number whose root is desired lies outside the range of any root column of the table, a guess must be made to be used as the initial approximation. Also in the case of very high numbers the decimal setup must be changed to allow proper entrance and transfer of numbers.

If the root of a decimal number is sought, the decimal point must be shifted according to the desired root, i.e., for the 6th root of a decimal number, the point must be shifted 6 places at a time to the right until the number falls into a range of the desired root column. The decimal point in the final root must be shifted to the left one place for each six-place shift.

general formula

A = approximation

B = the base number whose root is to be found

n = the desired root

$$\frac{\dfrac{A^n(n-1)+B}{A^{n-1}}}{n}$$

PROGRAM

Decimals:

Upper dials	5	Tab at 6
Keyboard	5	Slide at 5
Lower dials	10	Auto KB Clear to the right

Step 1. The initial approximation is raised to the nth power. Set the initial approximation on the keyboard and square it. Then transfer and multiply by the number on the keyboard n-2 times.

Step 2. Return Auto KB Clear to the left; lock the upper dials; place the lever in Non Entry position and clear the keyboard.

Step 3. Transfer and multiply by n-1.

Step 4. Add B to the number in the lower dials.

Step 5. Unlock the upper dials and place the lever in Entry position.

Step 6. Copy the number from the right upper dials to the keyboard and divide.

Step 7. Enter the number in the right upper dials as a dividend. Divide by n.

The result in the right upper dials is the next approximation of the base number. Repeat this cycle of seven steps, each time using the new approximation until two approximations coincide. The final root will be correct to five decimal places.

EXAMPLE

Find the $\sqrt[5]{1425.75}$

The initial approximation from the table is 4.25.

PROGRAM

Decimals:		
Upper dials	5	Tab at 6
Keyboard	5	Slide at 5
Lower dials	10	Auto KB Clear to the right.

Step 1. Square 4.25. Transfer and multiply three times (n-2=3). Result in the lower dials is 1386.57899; upper dials 326.25388.

Step 2. Return Auto KB Clear to the left; lock the upper dials; place the lever in Non Entry position and clear the keyboard.

Step 3. Transfer and multiply by 4 (n-1). Result in the lower dials is 5546.31596.

Step 4. Add 1425.75 to the lower dials.

Step 5. Unlock the upper dials and place the lever in Entry position.

Step 6. Copy 326.25388 from the right upper dials to the keyboard and divide.

Step 7. Enter 21.37006 from the upper right dials as a dividend. Divide by 5 (n).

Result in the upper dials is 4.27401 (second approximation of the 5th root of 1425.75).

Step 8. Repeat steps 1 through 7 using the second approximation 4.27401 to determine the third approximation of the 5th root of 1425.75.

Result in the upper dials is 4.27374 (third approximation).

Step 9. Repeat steps 1 through 7 using the third approximation 4.27374 to determine the fourth approximation of the 5th root of 1425.75.

Result in the upper dials is 4.27374 (fourth approximation).

The fourth approximation, 4.27374, coincides with the third approximation, thereby automatically proving that the third approximation is the 5th root of 1425.75.

$$\sqrt[5]{1425.75} = 4.27374$$

TABLE OF APPROXIMATIONS FOR HIGHER ROOTS

Initial Approximation	4th Root	5th Root	6th Root	7th Root	8th Root	9th Root	10th Root
	RANGES						
1.25	0-4	0-5	0-7	0-9	0-13	0-18	0-24
1.50	4-7	5-11	7-18	9-30	13-49	18-79	24-128
1.75	7-12	11-23	18-44	30-82	49-153	79-286	128-537
2.00	12-20	23-43	44-92	82-196	153-416	286-884	537-1878
2.25	20-32	43-76	92-180	196-426	416-1012	884-2404	1878-5710
2.50	32-48	76-125	180-327	426-859	1012-2254	2404-5918	5710-15534
2.75	48-68	125-196	327-565	859-1624	2254-4668	5918-13420	15534-38581
3.00	68-95	196-298	565-931	1624-2910	4668-9095	13420-28422	38581-90900
3.25	95-130	298-438	931-1478	2910-4988	9095-16834	28422-56815	
3.50	130-173	438-626	1478-2269	4988-8225	16834-29817	56815-97800	
3.75	173-226	626-874	2269-3386	8225-13119	29817-50836		
4.00	226-290	874-1194	3386-4927	13119-20322	50836-93900		
4.25	290-366	1194-1603	4927-7012	20322-30679			
4.50	366-458	1603-2116	7012-9787	30679-45267			
4.75	458-565	2116-2753	9787-13423	45267-65437			
5.00	565-690	2753-3536	13423-18120	65437-97700			
5.25	690-835	3536-4486	18120-24114				
5.50	835-1001	4486-5631	24114-31676				
5.75	1001-1191	5631-6999	31676-41119				
6.00	1191-1407	6999-8621	41119-52800				
6.25	1407-1652	8621-10529	52800-67125				
6.50	1652-1926	10529-12762	67125-84550				
6.75	1926-2234	12762-15359	84550-99500				
7.00	2234-2577	15359-18362					
7.25	2577-2958	18362-21818					
7.50	2958-3380	21818-25775					
7.75	3380-3846	25775-30287					
8.00	3846-4358	30287-35409					
8.25	4358-4920	35409-41203					
8.50	4920-5534	41203-47730					
8.75	5534-6204	47730-55061					
9.00	6204-6933	55061-63265					
9.25	6933-7725	63265-72420					
9.50	7725-8582	72420-82605					
9.75	8582-12000	82605-99400					

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